ANDOYER'S VERSION OF HANSEN'S METHOD

C.Altavista
(Observatorio Astronómico,La Plata)

This paper deals with Hansen's method for calculating perturbations. A brief outline is given of a comparison between Hansen's method in its original form and Andoyer's version of it, as contained in his "Cours de Mécanique Celeste" . The summary includes the main distinctive characters and similarities present in both procedures. A final remark gives the author's opinion on Andoyer's method.

A: Distinctive characters in Andoyer's treatment.

1.- Fixed reference plane.

2.- Quantities to be disturbed:

a: semi-major axis of the auxiliary ellipse.

l(or γ): mean anomaly

h_o: the constant or areas in the two body problem.

z: the ordinate respect to the fixed reference plane.

3.- The perturbation in mean anomaly is put as a correction to g_o, and defined by: $g = \mu t + g_o + \sigma$

4.- The perturbations in a and σ are calculated from the values of the perturbations in h_o and $-\frac{h_o}{a}$.

5.- The orthogonal components of the acceleration are developed in analytical form.

6.- There are three determining functions for calculating the perturbations in ω and β. These functions depend on the values of the perturbation in latitude, within small quantities of second order.

7.- The equation for the perturbation in latitude is a non homogeneous differential equation of second order.

8.- The disturbing function W has two parts:

a) The "proper" disturbing function V (which splits also in two parts).

b) A complementary part gives rise when the difference between the value of the mean motion defined by $\mu^2 a^3 = k^2$, should be put into coincidence with the observational value defined by: $g = \upsilon t + g_0$.

9.-Newcomb's method is used to develop the disturbing function, when its analytical development is to be considered.

10. Cauchy's method is suggested for numerical computation of perturbations. Harmonic analysis can then be applied.

11. The differences in the determination of the constants of integration depend on the form in which the absolute elements are defined.

12. Numerical values of those constants got in the first approximation are availed for calculating the increments (of these constants) in the second approximation.

13. No use is made of Hansen's theorem, according to which quantities that depend on the time and are out of the integral sign, can be put under the integral sign, and then the integrations can be performed taking a constant time for these quantities.

14. The determing function P_1 is arbitrary.

B: Similarities.

1.- An auxiliary ellipse is chosen on the fixed plane. There are four absolute elliptic constants in the plane. The other two constants determine the position of the fixed plane.

2.- The differential equation of motion are put in terms of the orthogonal components of the accelleration. It is understood that the values of the "reduced" radius-vectors must be taken in the formerly quoted equations.

3.- There is only a determining function to calculate the perturbations in latitude.

4.- The coefficients of the derivatives of the disturbing function are expressed, in both approximations, in terms of periodic series with argument g.

5.- The calculation of second order perturbations has two parts:

1) The first part depend directly on terms that contain the values of the first order perturbations. This part must be completed up to terms of second order.

2) The second part results from considering the variations of the determining functions. These variations arise from the influence of the perturbations of first order in these functions.

C: Remarks

It could be said that Andoyer's treatment of the statement of the problem is inferior to Hansen's original paper. There are several objections to bear in mind. Firstly there is one more quantity to be disturbed. Secondly, as Andoyer takes a fixed reference plane, he must express the disturbing function in terms of the "reduced" radius-vectors of the different bodies. New troubles arise when the difference between n and g is taken into account. For it, he is obliged to add a complementary part to the disturbing functions.

On the other hand the statement is such that there are three determining functions to calculate the perturbations in h_o and h_o/a, instead of only one as appears in Hansen's method. And we must also say that the suggestion for using Cauchy's method is not safe of criticism. The troubles are due to the fact that there are two expansions to be performed, instead of only one as it occurs in the ordinary case.

It can finally be said that the method seems to me -from the point of view of the analytical computations- very skillful but it is rather inadequate in its analytical statement.

This does not mean that some special devices of the method can not be used. Besides this the only tests are got when numerical applications are made. And of course it depend also on the case to be considered.

Special attention must be paid to the fact that there are denominators of second order in the first approximation.